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Critical behavior of the one-dimensional annihilation-fission process $2A \rightarrow \emptyset$, $2A \rightarrow 3A$

Géza Ódor
Research Institute for Technical Physics and Materials Science, P.O. Box 49, H-1525 Budapest, Hungary
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Numerical simulations and cluster mean-field approximations with coherent anomaly extrapolation show that the critical line of the $1d$ annihilation-fission process is separated into two regions. In both the small and high diffusion cases, the critical behavior is different from the well known universality classes of nonequilibrium phase transitions to absorbing states. The high diffusion region seems to be well described by the cyclically coupled directed percolation and annihilating random walk. Spreading exponents show nonuniversal behavior.

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Nonequilibrium phase transition may take place even in one-dimensional systems. However, the ordered phase lacks fluctuations that could destroy the state. If the system has evolved into that state it will be trapped there. We call this transition to an absorbing state. First order transitions among these are very rare; there have been only a few systems found that exhibit a discontinuous jump from the active to the absorbing phase [1–4].

Continuous phase transitions have been found to belong to a few universality classes, the most robust of which is the directed percolation (DP) class. According to the hypothesis of [5,6], all continuous phase transitions to a single absorbing state in homogeneous systems with short ranged interactions belong to this class provided there is no additional symmetry and quenched randomness present. The most prominent system exhibiting phase transition of this class is the branching and annihilating random walk with one offspring (BARW). Furthermore, systems with infinitely many absorbing states were also found to belong to this class [7–9].

An important exception from the DP class is the so-called parity-conserving (PC) class where particles follow a branching and annihilating random walk with two offspring (BARW2) $A \rightarrow 3A$, $2A \rightarrow \emptyset$ that conserve the parity of their number [10–13]. Particles following BARW2 may also appear as kinks between ordered domains in systems exhibiting two absorbing states [14–18]. However, it was realized that the BARW2 dynamics alone is not a sufficient condition for a transition in this class, but an exact Z_2 symmetry between the two absorbing states is necessary, too [17,19–22].

Recently, a study on the annihilation-fission (AF) process $2A \rightarrow \emptyset$, $2A \rightarrow 3A$ [23] suggested that neither the BARW2 dynamics nor the Z_2 symmetry but simply the occurrence of two absorbing states can result in a PC class transition. This model without the diffusion of single particles was introduced first by [7]. The renormalization group analysis of the corresponding bosonic field theory was given by [24]. This study predicted a non-DP class transition, but it could not tell to which universality class this transition really belongs.

A fermionic version of the AF process introduced in [23] is controlled by two parameters, namely, the probability of pair annihilation p and the probability of particle diffusion d. The dynamical rules are

$$\begin{aligned} &A\emptyset, \emptyset A A \rightarrow AAA \quad \text{with rate } (1-p)(1-d)/2, \\ &AA \rightarrow \emptyset\emptyset \quad \text{with rate } p(1-d), \\ &A\emptyset \leftrightarrow \emptyset A \quad \text{with rate } d. \end{aligned} \tag{1}$$

Carlon *et al.* suggested that the phase diagram can be separated into two regions. For low values of d (less than approximately 0.3), they found a continuous phase transition belonging to the PC universality class. For large values of d, however, a first order transition was reported. This claim is based on mean-field, pair mean-field, and density matrix renormalization group method (DMRG) [25] calculations. An even more recent preliminary numerical study [26] found non-PC classlike critical exponents and posed the question if there is a new type of nonequilibrium phase transition occurring here. In this paper I report more detailed numerical results: simulations and generalized mean-field (GMF) approximations with coherent anomaly (CAM) extrapolations that give numerical evidence of a rich phase diagram with two different kinds of universality classes in the low d and high d regions.

For simulations a parallel update version of the AF process was used since the model can effectively be mapped on a massively parallel processor ring [27]. To avoid collisions by synchronous updates, the lattice is divided in such a way that annihilation and exchange steps are done on two-sublattices with rates $p(1-d)$ and d, respectively, while creation is performed on three-sublattices with the rate $(1-p)(1-d)$. Thus in $d=1$ three-sublattices are defined as shown on Fig. 1 and updated in a circular way $(A,B,C,A,\ldots)$. This algorithm is faster than the random sequential one because we do not have to generate the random sites. The critical point has been determined for $d=0.05,0.1,0.2,0.5,0.9$ by following the decay of the particles from a uniform, random initial state $(\rho(t) \propto t^{-\delta})$. The local slopes curve of the density

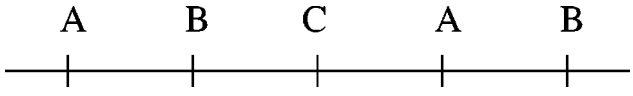

FIG. 1. Three sublattices.

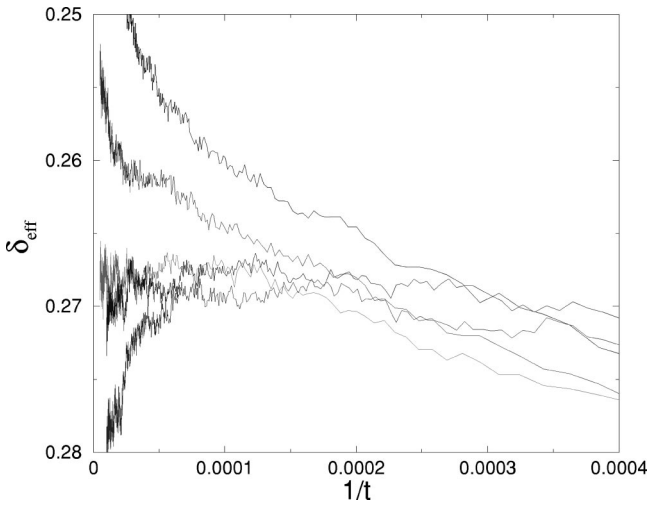

FIG. 2. Local slopes of the particle density decay in the parallel update model at $d=0.2$. Different curves correspond to 0.248 07, 0.248 05, 0.248 02, 0.248, 0.247 95 (from bottom to top). The simulations were performed on a ring of size $L=24\,000$ up to 4×10^5 MCS. Throughout the whole paper t is measured in units of Monte Carlo sweeps (MCS).

$$\delta_{eff}(t)=\frac{-\ln[\rho(t)/\rho(t/m)]}{\ln(m)} \tag{2}$$

(where we usually use $m=8$) at the critical point goes to exponent δ by a straight line, while in subcritical or supercritical cases it veers down (up), respectively. As Fig. 2 shows at $d=0.2$ and $p=0.248\,02$, this quantity scales without any relevant corrections (i.e., the straight line is horizontal) and in the $1/t\to0$ limit goes to $\delta=0.268(2)$.

Very similar results have been obtained for $d=0.05$ and $d=0.1$, showing that the exponent δ is close to the corresponding value of the PC class [0.285(10) [11]], but appears to be significantly smaller. This δ is in agreement within error margin with the value [$\sim0.272(18)$] one can obtain from DMGR calculations [23] using scaling laws $\delta=\beta/\nu_{||}=\beta/\nu_{\perp}/z$.

For stronger diffusion rates $d=0.5$ and $d=0.9$ one can observe slower density decays with exponent $\delta=0.215(10)$ (Fig. 3). Note that I checked that changing the abscissa on the local slopes graphs from $1/t$ to $\ln(t)/t$ does not eliminate the differences between small and large d behaviors. Also by assuming strong correction to scaling I could not get coherent estimates for β and δ that would be in agreement with DP or PC classes. This value of δ is in agreement with that of the coupled DP plus annihilation random walk suggested by [28]. This model was proposed to explain the space-time behavior of the AF with the help of a multicomponent model, where A particles (corresponding to pairs in AF) perform BARW processes $A\to2A$, $A\to\emptyset$ and occasionally create B particles (corresponding to lonely particles in AF) who follow random walk plus annihilation: $2B\to A$ (ARW). The space-time evolution picture of this model looks similar to that of AF (and very different from that of BARW2) but the numerical simulations predicted somewhat different exponents as those of the AF [23,26,29,30]. By approaching $d=1$, corrections to scaling are getting stronger and this might lead to the conclusion of [23]—that for $d>0.3$ the transition

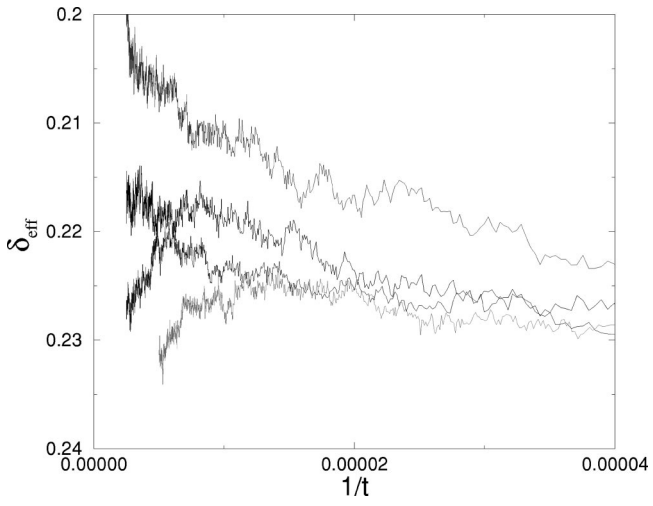

FIG. 3. The same as Fig. 2 at $d=0.5$. Different curves correspond to $p=0.2796$, 0.279 58, 0.279 55, 0.2795 (from bottom to top).

is of first order. The present study cannot confirm those predictions based on extrapolations to DMRG results of small system sizes ($L<100$). The first order transition for $d>0.3$ appears to be unlikely also because the mean-field approximation of the model exhibits a continuous phase transition (with $\beta_{\rm MF}=1$) and that corresponds to the strong diffusion limit without fluctuations.

The differences from PC class and from first order transition can be seen even more clearly by direct measurements of the order parameter β exponent ($\rho(\infty)\propto\epsilon^{\beta}$). By looking at the effective exponent defined as

$$\beta_{eff}(\epsilon_i)=\frac{\ln\rho(\epsilon_i)-\ln\rho(\epsilon_{i-1})}{\ln\epsilon_i-\ln\epsilon_{i-1}}, \tag{3}$$

on Fig. 4 one can observe two regions again: for $d<0.3$ the

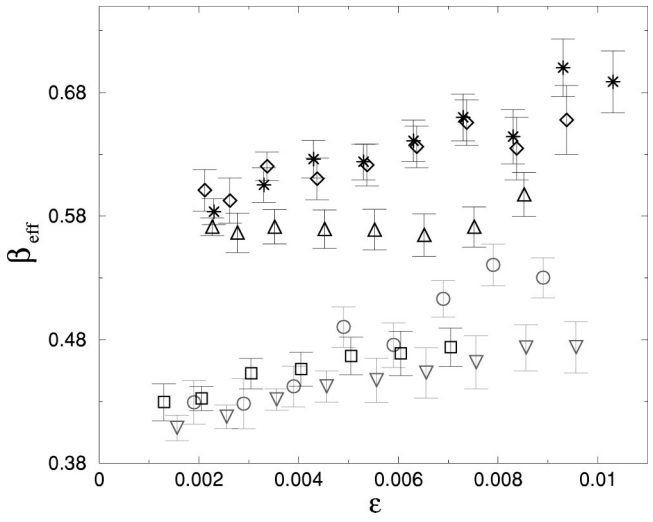

FIG. 4. Order parameter exponent results of the AF model for $d=0.9$ (circles), $d=0.5$ (squares), $d=0.2$ (diamonds), $d=0.1$ (triangles), $d=0.05$ (stars). The simulations were performed on a ring of $L=24\,000$ sites such that averaging was done following the steady state reached for 1000 samples

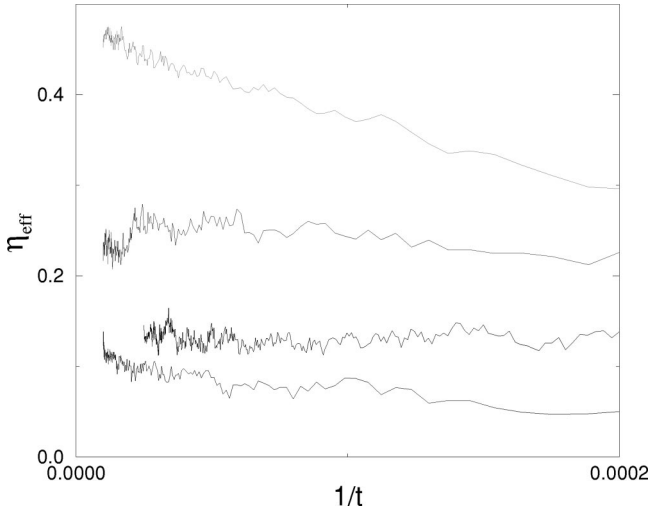

FIG. 5. Local slopes of the particle number in seed simulations in the parallel update model at $d=0.05$, $d=0.2$, $d=0.5$, and $d=0.9$ (from bottom to top). The simulations were performed on a lattice of size $L=24\,000$ up to 10^5 MCS.

β_{eff} tends to 0.58(2) as $\epsilon=(p_c-p)\to 0$, while for $d>0.3$ it goes to $\sim 0.4(2)$.

For small diffusion rates the value 0.58 is very different from that of the PC class [0.94(10)] [11,22]) and from that of DP class [0.2765(1)] [6], but is in agreement with preliminary simulation results of [26] and [30]. For strong diffusion it coincides fairly well with the value 0.38(6) that was found for the coupled DP+ARW model [28].

The results for the order parameter exponent have been verified by generalized mean-field approximations [31,32] with coherent anomaly extrapolation [33]. This method has been proven to give precise estimates for the DP [34] and PC [2] classes. The details of the method are described in [22] and a forthcoming paper [35] will discuss the results and show estimates for other exponents as well.

For technical reasons a variant of the AF model was investigated where the creation is symmetric: $A\emptyset A\to AAA$. This does not change the essence since the $AA\emptyset/\emptyset AA\to AAA$ process can be decomposed into a $A\emptyset\to\emptyset A$ exchange and a symmetric creation. One can easily check that the mean-field approximation ($N=1$) and its result are trivially the same as that for model 1, namely,

$$\rho(\infty)=(1-3p)/(1-p). \tag{4}$$

Higher order approximations for $N=2,3,\ldots,7$ converge to the simulation results of this variant (Fig. 5) and the phase diagram qualitatively agrees with that of Ref. [23]. The leading order singularity at critical point remains mean-field type, i.e., $\beta_{\mathrm{GMF}}=1$. Furthermore, dynamical MC simulation of this variant gives the same critical decay behavior as the original AF model. According to CAM, the amplitudes of these singularities $a(N)$ scale in such a way that

$$a(N)\propto(p_c(N)-p_c)^{\beta-\beta_{\mathrm{MF}}} \tag{5}$$

and the exponent of true singular behavior, can be estimated. The CAM extrapolation has been applied for the highest levels of approximations ($N=4,5,6,7$) and resulted in β_{CAM} values in agreement with those of simulations (see summarizing Table I).

TABLE I. Summary of results. The nonuniversal critical parameter p_c of the parallel model is shown here.

d	0.05	0.1	0.2	0.5	0.9
p_c	0.250 78	0.248 89	0.248 02	0.279 55	0.4324
β_{CAM}		0.58(6)	0.58(2)	0.42(4)	
β	0.57(2)	0.58(1)	0.58(1)	0.40(2)	0.39(2)
δ	0.273(2)	0.275(4)	0.268(2)	0.21(1)	0.20(1)
η	0.10(2)		0.14(1)	0.23(2)	0.48(1)
δ'	0.004(6)		0.004(6)	0.008(9)	0.01(1)

Note that the CAM extrapolation does not work for $d>0.5$ and $d<0.1$, because corrections to scaling are getting strong, and it is very difficult to find the steady state solutions of the GMF equations for $N>7$. Time dependent simulations from a single active seed for these models have been proven to be a very efficient method [36] since the slowly spreading clusters do not exceed the allowed system sizes and hence do not feel finite size effects. Very precise estimates for the cluster ''mass'' exponent $N(t)\propto t^{\eta}$ and the cluster survival probability exponent $P(t)\propto t^{-\delta'}$ have been obtained in DP and PC classes (see references in [37]). However, for example, in the case of systems with infinitely many absorbing states these exponents seem to depend on initial conditions [7–9]. A rigorous explanation is still missing. The simulation results for the cluster mass (Fig. 6) show strong dependence of η on d. For $d=0.05$ and $d=0.2$, η is close to 0.15, while it increases to $\eta=0.23(2)$ for $d=0.5$ and reaches $\eta=0.48(1)$ for $d=0.9$. The survival probability exponent δ' is roughly zero, expressing the fact that a single particle survives with finite probability (see Table I).

The continuously changing exponent η is in agreement with the preliminary observations of [26]; however, β and the δ seem to be constant in the region $0.05\leqslant d\leqslant 0.2$ and

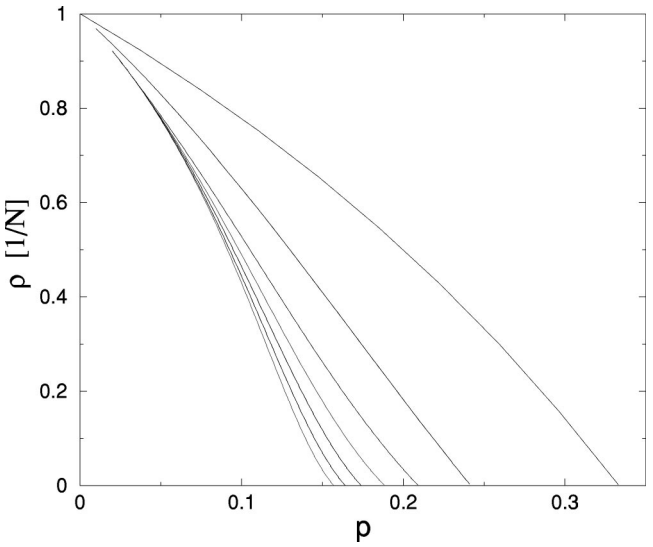

FIG. 6. GMF approximations of the particle density in the symmetrized AF model at $d=0.5$ for $N=1,2,\ldots,7$ (from right to left curves).

$0.5 \leqslant d \leqslant 0.9$, suggesting two different universality classes. The nonuniversal behavior of the cluster exponent may not be so surprising since it was also observed in the $d=0$ limit of the AF in the pair contact process [7–9].

In conclusion, numerical simulations and analytical GMF +CAM calculations suggest two different universality classes in the AF model along the phase transition line. For small diffusion, single particles cannot escape clusters where pair annihilation and creation dominates. For larger diffusions these single particles will wander among pair creation-annihilation clusters and the coupled BARW+ARW description of [28] will be valid. These unknown universal behaviors may sound uneasy following two decades when the DP hypothesis has been found to describe almost every continuous transition to absorbing state, but they do not contradict it at all. In the AF model the absorbing state is not singlet, there are two (nonsymmetric) absorbing states where the system can evolve with equal probabilities. One of them is not completely frozen, but a single particle diffuses in it.My results do not support the conclusions of [23] for a PC class transition without a BARW2 process and Z_2 symmetry nor the first order transition for large d, but the exponents do not contradict those of [23] within numerical precision. It is likely that the two exponent ratios determined by [23] are close to those of PC class accidentally. The cluster exponent η behaves nonuniversally like in the pair contact process, the $d=0$ limit of the AF. Deeper understanding of this model that would reveal hidden symmetries responsible for the strange critical behavior of AF would be highly desirable.

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